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FINAL REPORT

PROPELLER/ICE INTERACTION: AN INVESTIGATION INTO THE PROBLEM OF PREDICTING PROPELLER/ICE INTERACTION FORCES

TP 4022E

December 1982

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17 Abstract

An investigation into the problem of predicting propeller/ice interaction forces, and in particular, the application of the finite element method to the problem, is presented. The study identifies current practices, previous efforts involving the use of the finite element method in related problems, and leading workers in the field of ice mechanics. Currently, simplified approaches are used (eg. modified Jagodkin formulation). These approaches incorporate simple ice failure modes which neglect to consider the complex triaxial ice stress fields developed within the ice block during milling. If a more sophisticated formulation of the propeller/ice interaction process is to be developed, a more realistic mathematical model of the ice block is required. The finite element method seems to be the most appropriate means of doing this. Consequently, an approach which involves modelling the ice block by finite elements is proposed. Anticipated problems with regard to ice mechanics and nonlinearities are discussed. Recommendations for further work are given.

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17 Résumé

Une étude se penche sur le problême de l'interaction entre l'hélice d'un navire et la glace, et en particulier sur l'application de la méthode d'eléments finis dans ce contexte. Une revue présente les méthodes courantes, les applications faisant usage d'eléments finis et les auteurs principaux dans le domaine de la mécanique des glaces. Jusquà date, les analyses se limitent à des approximations simples (eg. la technique modifiée de Jagodkin), qui négligent la distribution tridimensionelle du stress, à l'interieur d'un bloque de glace pendant le broyage. Afin de mieux simuler l'interaction entre hélice et glace, un modèle mathématique plus réaliste doit être mis au point pour représenter le bloque de glace. On suggère pour ceci l'usage de la méthode d'éléments finis, qui semble tout a fait appropriée. On discute les problèmes aux quelles on peut s'attendre, et ayant attrait à la nonlinéarité et aux propriétés mécaniques particulières de la glace. Enfin, des recommendations concernant la direction future de travaux dans ce domaine sont présentées.

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1. Introduction

This report describes work carried out under DSS/TDC Contract No. 1SD81-00129 on the stress analysis of modern icebreaker propellers subjected to propeller/ice interaction forces. The aim has been to employ the finite element of structural analysis to this complex problem, with the objective of developing a reliable technique to be used in establishing ice class rules which will enable effective design and regulation of propellers for Arctic service.

The work plan for the project has been divided into two parts, i.e. (i) a parametric study of propellers operating in an ice environment using simplified loading definitions, and (ii) an investigation into the problem of predicting propeller/ice interaction forces, considering stress and fracture patterns in the ice milling process. This report describes the work relating to the problem of predicting propeller/ice interaction forces; the work relating to the parametric study is presented in another report (Reference 1).

The primary objective of this part of the project has been to conduct a feasibility study on the application of the finite element method for predicting propeller/ice interaction forces. This involves modelling the ice block by finite elements to determine stresses in the ice block due to penetration of the propeller blade, which will then be used to define the loads acting on the blade. Secondary objectives include a review and assessment of existing methods for predicting propeller/ice interaction forces, and more realistic (but still simple) loading distribution for use in the parametric study.

The investigation began by conducting a state-of-the art review. The purposes of this review were to:

i. Identify current practices for predicting propeller/ice interaction forces;

ii. Identify previous efforts involving the use of the finite element method in ice milling and/or related problems; and

iii. Identify leading workers in the field of ice mechanics.

Following the state-of-the art review, an overall assessment of existing methods for predicting propeller/ice interaction forces was made. Then the application of the finite element method as an alternative to existing melthods was considered in detail. Finally, a course of action regarding further work required was established.

The assessment of existing methods for predicting propeller/ice interacting forces is given in Section 2. An investigation into the feasibility of applying the finite element method as a means of solving the problem is discussed in Section 3. The recommended course of action for further work is described in Section 4.

Three centres involved in studies relevant to this investigation have been visted - Laval University, Quebec; Ecole Polytechnique, Montreal; and the National Research Council, Ottawa. Trip reports are provided in the Appendix.

2. Review of Existing Methods for Predicting Propeller/Ice Interaction Forces

Regulations for ice class propellers as set down by the various classification societies are based on unsophisticated methods for handling propeller/ice interaction forces. These methods are basically empirical in nature (eg. basing the design on a percentage increase over hydrodynamic requirements, or using an ice torque based on coefficients specified by the particular society) and do not attempt to take into account the actual mechanics of the milling process. These methods are easily incorporated when using conventional methods for analyzing propellers. However, they are not adequate for the more sophisticated finite elemlent method of analysis, where realistic formulations of propeller/ice interaction forces are required, if full potential of the method is to be realized. Furthermore, society, regulations (with the exception of those used by the Soviet Register of Shipping) are based on sub-Arctic operations, and are unreliable for Arctic service.

In recent years, various attempts at realistically representing the propeller/ice interaction forces for use in finite element analyses of propellers have been made. Allis Chalmers Corp. [2] did a life expectancy analysis of an icebreaker blade subjected to the fatigue loading imposed on it by ice milling operations. For a particular cutting depth, the passage of the blade through the ice was described by using seven uniformly distributed pressure loading conditions as shown in Figure 1. An assumed ice-crushing pressure was used. Gabel et al., [3], formulated a more detailed description of blade loading. They used a modified Jagodkin approach to describe the blade/ice loads, and included they hydrodynamic forces as well. Taking into consideration the angle of advance, i.e. the relataive values of ship speed and propeller rotation rate, the model was able to incorporate three modes of ice failure - crushing, slicing and shearing, as shown in Figure 2. Variations in blade thickness, pitch angle and chord length along blade radius are accounted for. Unfortunately, the blade is assumed to be loaded from a line of constant radius to the tip, thereby prohibiting an analysis of the passage of blade through the ice, as was done in Reference 2. Ice crushing and shearing strengths must be supplied to the model as input data.

The investigation has failed to identify anything (in the open literature) more advanced than the approach used by Gabel et al. However, we have contacted a U.S. consultant who appears to have taken the blade/ice loading somewhat further. The exact nature of this work was not determined, since it is currently proprietory. Further discussions with this consultant in an attempt to determine more information on the work are warranted.

3. Application of the Finite Element Method to Ice Milling

3.1 Introduction

The assumption that propeller/ice interaction forces due to ice milling are explicity related to the basic ice strength data (as in the models examined in Section 2) is an oversimplification. Complex triaxial ice stress fields develop within the ice block during milling, and identification of the corresponding propeller loads, therefore, requires a more detailed investigation of ice failure modes. Furthermore, as the propeller blade passes through the ice, the area of the blade in contact with the ice experiences significant geometric changes. This area can vary from being wedge shaped to being a warped surface. The simple models used in practice cannot take this complex process into account either.

If an accurate formulation of the ice milling is to be developed, it appears that the most logical approach is to mathematically model the ice block. The finite element method seems to be the most appropriate means of doing this. It has already been successfully applied to a large variety of ice mechanics problems including strength analysis, buckling, elasto-plastic analysis, creep and crack propagation [4-17]. Although an exhaustive literature search has failed to turn up any publications specifically on the analysis of ice milling by the finite element method, the method has been used to a limited extent in similar metal machining processes [18, 19] suggesting it probably can be applied to ice milling as well.

The milling constitutes a nonlinear process. Well established incremental procedures are available for solving nonlinear structural problems [20-22]. The basic solution scheme for ice milling is expected to be much the same. However, as certain important additional characteristics unique to ice milling remain to be incorporated, considerable further development work both in the form of experimental investigations and extensions or modifications of existing numerical algorithms is nonetheless necessary. The special requirements of an ice milling analysis are identified and discussed below.

3.2 Basic Mechanical Property Definition

Several factors affect the mechanical properties of ice including grain size and orientation, salinity, temperature and loading rate. Not surprisingly, therefore, significant variations in mechanical properties exist and a large amount of ice testing is required to quantify the effect of each factor. Considering that interest in ice mechanics has developed relatively recently from hydrocarbon exploration activities in the Arctic, shortages of ice data of sufficient quality to define ice mechanical properties in the detail required by sophisticated numerical methods are not surprising.

Testing procedures continue to be refined thereby frustrating attempts to catalogue the mechanical properties of the various ice types. Until recently the uniaxial unconstrained compressive strength of ice was generally reported in literature to have a strain-rate dependence similar

to that in Figure 3. After reaching a maximum for a strain rate of order $10^{-3}sec^{-1}$, crushing strength decreased as strain rate increased towards $10^{-2}sec^{-1}$ or $10^{-1}sec^{-1}$ and thereafter strength remained constant. Observing that significant internal cracking activity and failure by brittle fracture (elastic response up to the point of fracture) accompanied high loading rates prompted Dr. N.K. Sinha of the Division of Building Research at the National Research Council to investigate cracking activity occurring during tests at high rates. Failure strengths were found to be extremely susceptable to initial microcracking caused by specimen end roughness. Even small imperfections in the end finish produced stress concentrations which served as crack initiation sites. Through refinements in techniques for specimen end finishing including smoothing with fine emery paper and polishing with alcohol-moistened tissue paper, significant increases in strength have been achieved so that only small strength reductions now accompany high loading rates [23]. Compare dashed lines on Figure 4.

Acoustic sensing devices employed during specimen testing provided further evidence to support Sinha's hypothesis regarding sensitivity of specimen strength to end imperfections. Two acoustical sensors appropriately attached to the specimen enabled the source of acoustic signals associated with cracking activity to be pinpointed accurately. As expected, signals were observed to originate predominately from platen ends during early stages of specimen loading. Dr. Sinha is currently investigating new specimen end configurations as a means of further reducing strength sensitivity to end imperfections when loading rates are high.

In recent discussions, Dr. Frederking [24] expressed other reservations concerning ice behaviour at high loading rates. In particular, he is still not convinced that mechanical strengths and fracturing modes will remain unchanged as loading rates are increased above $10^{-1}sec^{-1}$ and as inertial forces arising from material disposal assumed increased importance. Controlled experiments although difficult to perform at high strain rates are obviously necessary to ascertain the extent to which ice impact behaviour differs from brittle behaviour.

3.3 Failure Envelopes

The failure envelope, or alternately yield surface according to structural mechanics terminology, defines the limits of elastic material behaviour. Stresses outside the failure envelope cannot exist. Those on the envelope represent the onset of fracture or alternately plastic yielding (plastic strain increments being related to stresses by what is termed a flow rule) depending upon whether the material behaviour is brittle or ductile.

Considerable uncertainty in ice fracture predictions for multiaxial stress situations may develop from the unreliability of the failure envelope definitions associated with the scarcity of accurate multiaxial ice strength measurements. Even in biaxial situation, testing has been limited until recently to the plane strain tests of Frederking [17, 18]. Various

investigators have nonetheless generated two dimensional failure envelopes for ice by interpolating suitable curves to three or four uniaxial strength data points. Two factors, however, impose severe limitations on the accuracy of the envelopes generated in this way. Typical uniaxial fracture strengths in compression, σ_c^*, are between two and three times the corresponding uniaxial fracture strength in tension, σ_t^*. Also a high degree of uncertainty generally exists in the σ_c^* and σ_t^* data points due to significant scatter in experimental data. The strong dependence of the ellipticity of the failure envelope on the data points used thus implies an even larger uncertainty in the shape of the interpolated failure envelope particularly along the major axis defined by $\sigma_1 = \sigma_2$ as demonstrated in Figure 5. Increased reliability should follow from the use of some biaxial strength data in envelope definition [26-29].

A further shortcoming of curved fitted failure envelopes recognized by Michel [30] is that an important discontinuity between compressive and tensile states on the stress plane $\sigma_1 = -\sigma_2$ is lacking. Under tensile loadings, a maximum principal stress criterion applies and due to the extremely brittle nature of ice, the first microcrack propagates immediately through the whole specimen. The uniaxial tensile strength of polycrystalline ice under brittle conditions is defined by [31].

$$\sigma_t = 2\sqrt{\frac{6\sigma_{cv}\, G_{66}}{d^*}\left(1 - \frac{G_{66}}{E_{33}}\right)}$$

where

σ_{cv} is the surface energy
G_{66} is the shear modulus in plane perpendicular to c-axis [p2, 32]
E_{33} is Young's modulus along c-axis
d^* is diameter of equivalent spherical crystal

Essentially it differs from the uniaxial compressive strength [31]

$$\sigma_c^* = 2\left[C(\theta) + \left(\frac{6\sigma_{cv}\, G_{66}}{d^*}\right)\right]$$

by the cohesive constant, C, which depends strongly on temperature θ. This accounts for the discontinuity between tensile and compressive strengths on planes of maximum stress as evident in the failure criterion of Figure 6 proposed by Michel.

The failure envelope corners generated by introducing such discontinuities on the other hand generally implies a certain degree of arbitrariness in the definition of the flow rules associated with plastic deformation [33]. The selection of the corner flow rule may be critical to the determination of correct displacements and stresses.

Triaxial yield envelopes pose even greater difficulties. Until very recently little triaxial strength testing was conducted and since those tests were performed on a standard triaxial cell, applied stress ratios

were subject to the condition $\sigma_1 \neq \sigma_2 = \sigma_3$ [34, 35]. Jones used this device to define the failure envelope of Figure 7 for fine grained ice of glaciers. Hydrostatic pressures of order 15 to 20 atmospheres encountered at glacier beds were considered.

A new multi-axial testing machine with an electronic closed loop control system and brush type loading platens was recently used by F.U. Hausler [36] to perform extensive triaxial ice strength tests. An increased number of stress ratios available through the above-mentioned improvements in test equipment permitted a facetted failure surface to be defined. Although brush type platens were adopted with the aim of minimizing lateral strain constraints on the loaded ice faces, some concern has been expressed by Dr. Sinha that stress concentrations around the bristle edges induce premature cracking activity which reduces strength data obtained.

Arctec Canada Ltd. has performed a large amount of ice testing for a client. Although collected data is currently proprietary, under the contractual arrangements public release is possible after December 1982. Apparently Transport Canada has obtained rights to the data and Martec as consultants to Transport Canada may be able to gain access. The Ship Safety Branch of Transport Canada has not yet been approached regarding this matter.

Another potential source of triaxial data might be Laval University. Arctic ice investigations by Jean-Paul Nadreau under the direction of Professor Michel are aimed at identifying its creep properties and defining its triaxial failure envelope.

A serious deficiency of all triaxial ice testing is that tests are limited to the compressive octant of stress space. Although multiaxial tensile tests can in principle be conducted and the failure envelope fully defined, the problem of platen-to-specimen adhesion has not yet solved for ice.

3.4 Crack Propagation

For high loading rates typical of those encountered in ice milling by propellers, ice is brittle and it is vital that crack propagation be suitably accounted for in ice milling analysis. In linear fracture mechanics, the fundamental material property used to predict fracture is fracture toughness. Material resistance to fracture is related to the stress state at a defect such as a crack-tip when it begins to grow. For instance since the stress field in the vicinity of a crack-tip can be expressed in terms of parameter K called the stress intensity factor, i.e.

$$\sigma_{ij} = K\, f_{ij}(r,\theta)$$

the critical value, K_c, may be used to identify the onset of fracture. The plane-strain fracture toughness of a material is K_{Ic} where suffix I refers to tensile opening mode of crack extension as distinct from shear on anti-plane-shear modes [94]. If with cracking the load decreases and stress intensity factor becomes low enough, crack propagation is arrested

and K_{Ia} is then fracture toughness at arrest. For nonlinear fracture mechanics, plastic deformations at the crack-tip cannot be ignored and the J-contour integral is used as a crack extension criterion [37].

Little fracture toughness testing of ice has been conducted to date [38-43]. Typically ice toughness demonstrates a strong strain rate dependence as shown in Figure 8. Another interesting general observation is that ice fracture toughness reductions of 50% may result just from the crack being wet. Wet milling is, therefore, expected to differ significantly from dry milling.

Finite element algorithms for fracture propagation in metal and rock applications are well established. Special crack-tip elements, characterized by their shape functions including functions with stress singularities of order $r^{-\frac{1}{2}}$ as appropriate for crack-tip regions have been developed [44-48]. Stress intensity factors as generalized element degrees of freedom are determined directly from the solution of the equilibrium equations. An alternative to crack-tip elements involves adopting special geometric configurations of certain elements usually with isoparametric formulations to produce singularities of the required form at an element node [49-58]. The quarter-point technique is one well known example.

A typical algorithm predicting crack increments and their orientations for incrementally increasing loads using energy release concepts has been outlined by Ingraffea [59]. As the crack-tip advances, additional model nodes are usually introduced on the crack faces due to node dissociation behind the crack front (when an element interface separates, element nodes along the interface are disconnected and two model nodes are required at each point where one existed originally). If the total number of model nodes increases in consequence, the stiffness matrix bandwidth will expand significantly unless nodes are renumbered throughout. An economical alternative proposed by Ingraffea is the node grafting method [60] which exploits the fact that hierarchical formulations [61-66] of higher order isoparametric elements have variable numbers of nodes which may be removed and grafted as required along the crack while preserving the total number of model nodes and elements as well as a reasonable bandwidth. Crack arrest, branching and closure phenomena must also sometimes be accounted for [67-70].

Crack-tracking computations are costly and the number of cracks which can be analyzed economically is therefore severely limited. For concrete structures or rock where high density crack distributions are common, the adoption of a material damage approach to finite element fracture analysis is preferred [71-73]. Elements through which cracks propagate have their stiffness matrices modified to simulate the presence of a uniform distribution of microcracks. For instance with increasing crack density, the ability of the element to carry tension normal to crack orientation is reduced. Various fracture mechanics propagation criteria including energy release may still be used to control growth of the cracked region. The distributed crack damage concept has been proven to be very effective and economical, largely because of the algorithmic

simplicity associated with its use. Note that no topological modification of the finite element mesh is required and no singular elements are used. One shortcoming, however, is that element microcracks are unidirectional. The damage approach has been equally successful in creep rupture of metals where the damage is manifested as voids rather than cracks [74-76].

Due to the prominence of material cracking and fragmentation in ice milling, little doubt exists about the need for a damage model for ice. High loading rates associated with milling increase the importance of dynamic fracture considerations [77] which should be suitably accounted for within the model. Stanford Research Institute has developed a powerful damage modelling computer program describing either ductile and brittle fracture associated with impact loadings [78-80] which can be readily inserted in two-dimensional Lagrangian wave propagation programs such as the finite difference programs HEMP and PISCES. Their program accounts for nucleation and subsequent growth of cracks or voids as a function of stress level and stress duration, coalescence of cracks to form fragments, and the possible eventual separation of fragments. Numerous crack orientations are permitted within each element and the effects of inherent material flaws and faults can also be accounted for. The program has been refined to the point that even size of fragments can be reliably predicted.

While the Stanford Research Institute damage model appears to hold considerable potential as the basis of a damage model for ice, the requirement for further development work is nonetheless indicated. Microfracture nucleation and growth laws are empirical in that observational assessments of microfracture impact damage for several different types of material when correlated with loading history yielded the following expressions:

$$\dot{N} = N_0 \exp \frac{\sigma - \sigma_{no}}{\sigma_1} \qquad \dot{R} = \frac{\sigma - \sigma_{go}}{4\eta} R$$

where the nucleation rate $\dot{N}$ varies exponentially on the tensile stress σ, and the growth rate $\dot{R}$ is a linear function of stress and the size of microfracture R. The remaining parameters constitute material fracture properties and are to be established by curve fitting to data obtained for ice. In fact, the relevance of the above expressions to ice ought to be reassessed considering the decreased loading rates associated with ice milling and the uniqueness of ice properties attributed to high homologous temperatures.

3.5 Crushed Ice

High contact pressures under indentors tends to crush ice to a powder-like consistency and when crushing extends a significant distance into the ice block, the constitutive relations used to describe the material can significantly affect the accuracy of numerical models. Properties of ice after crushing are quite distinct from those of the original ice.

Crushed ice carries no tension and minimal shear but still provides resistance to hydrostatic pressure. The apparent similarity of its behaviour to that of soils or granulated rock suggests a Colomb-Mohr type of failure criterion (with tension cutoffs)[81, 82].

4. Conclusions and Recommendations for Future Work

After examining the problem of predicting propeller/ice interaction forces, it has been concluded that while current analysis and design procedures are oversimplified and inadequate, nonlinear finite element methodology appears to be sufficiently advanced that its application to the problem will estimate propeller loadings with required detail and reliability provided a complete description of ice properties is available. A continuation of this study is therefore recommended.

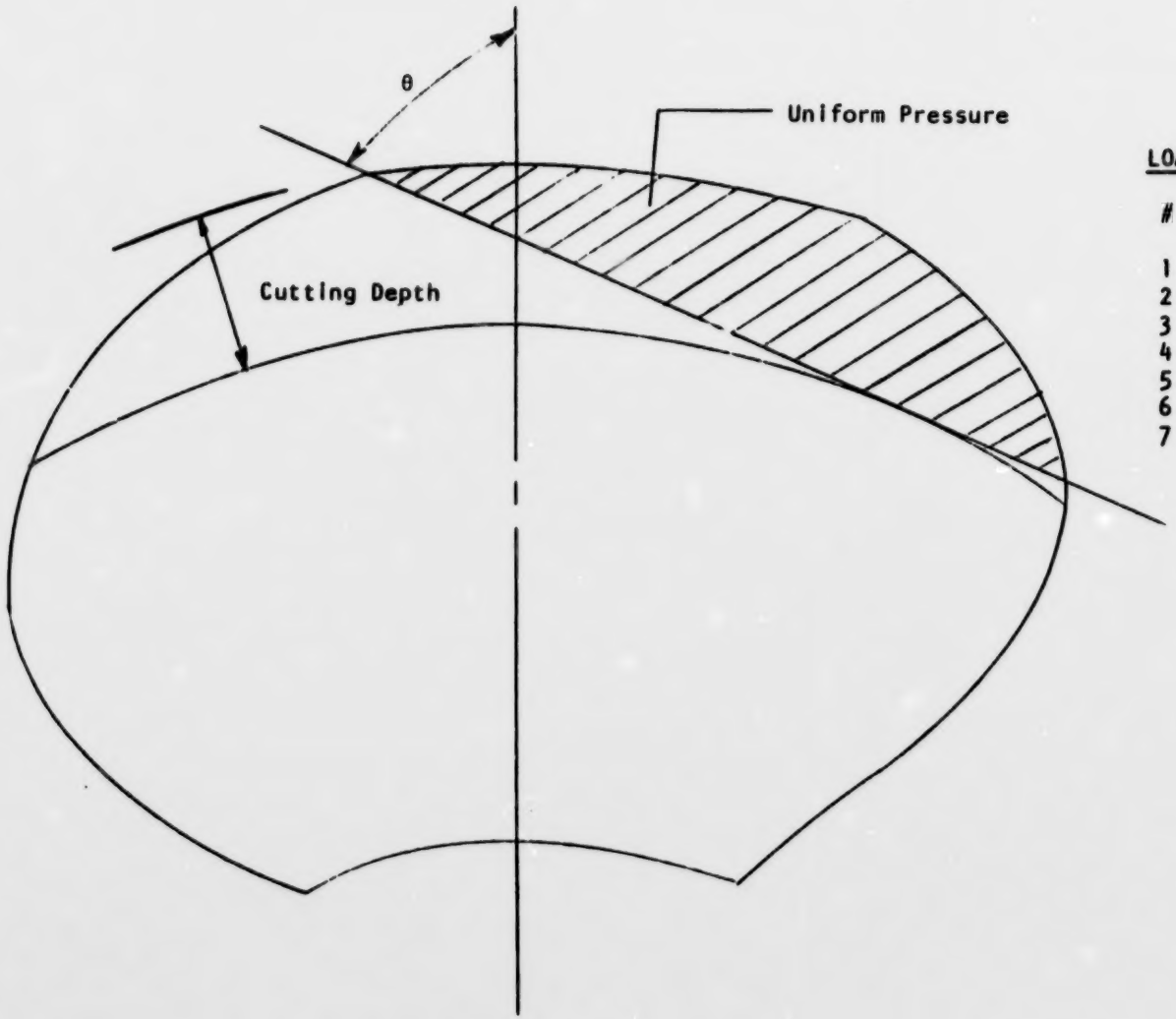
The true propeller milling situation involves complex structure-ice interactions. Ice pressures experienced by the propeller during milling affect propeller speed and deflections which in turn influence cutting parameters and thus ice pressures. Some approximations are necessary, for purposes of analysis, if analysis costs are not to become prohibitive. Determining the propeller response to computed ice pressure loads involves only the straightforward application of well established procedures of the finite element method and during development of an ice milling analysis capability, the propeller should therefore be assumed to be rigid and the influence of propeller deformation on the ice milling process ignored. Variations in propeller speed which can be determined from a detailed investigation of the dynamics of the ship propulsion system should similarly be ignored. Inspite of the assumptions that the propeller is rigid and rotates at constant speed, a rather complicated three-dimensional milling problem nonetheless remains for investigation. Since no previous application of analytical techniques such as the finite element method to any ice milling type situations involving continuous material fracture has been reported in open literature, it seems important that the feasibility of applying the developed analysis method to ice milling in two dimensions be demonstrated conclusively before attempting an analysis of three dimensional milling.

A major development effort will be required to produce a reliable algorithm for the analysis of even the simplified two dimensional milling problem. The iterative nature of solution algorithms for nonlinear problems makes them costly to execute and attention to coding efficiency is most critical. Utilization of commercial programs appears to be significantly more cost effective. Several commercial finite element programs including ADINA [83], MARC [84-86], DYNA2D [87] and DYNA3D [88] feature non-linear analysis capabilities suitable for milling applications. Similarly two nonlinear finite difference programs called PISCES [89] and HEMP [90] have considerable potential. Although the programs may be used at software bureaus for a modest royalty charge in addition to the normal fees for computer resources utilized, a more attractive option involves purchasing source listings from the program vendor and implementing modifications to provide special ice milling features such as a fracture damage model. Although further work is still required to evaluate the above options with the aim of identifying relative program merits, establishing required modifications and estimating the costs involved, tentative plans for the next study phase involve applying one of the above programs to simplified ice milling and perhaps studying the sensitivity of milling to ice properties, i.e. to perturbations of the

failure envelope, to adjustments of the damage model and to refinements in the crushed ice model.

Some experimental work is also desirable. Analytical results from the numerical models should be verified by comparison to experimental results from controlled two dimensional milling tests conducted specially for this purpose. Pressure tranducers may be installed on the tool pressure surface to establish the pressure distribution along the ice-tool interface. Ice fracture patterns should also be identified and compared with the predictions of the model. Scale of experiments should compare with that involved in the real propeller milling situation to ensure that similar fracture mechanisms are involved. Scale reductions of the order associated with the use of standard workshop metal milling machines in experiments [91] are unacceptable. Cooperation with an institution possessing an ice testing facility such as Ecole Polytechnique or Laval University will have to be investigated when planning a test program.

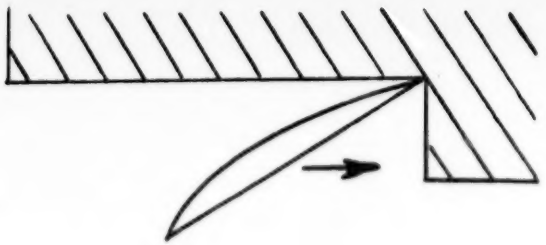
Although, the application of the finite element method as a means of handling the ice milling process will be much more accurate and complete than any simplified formulation, the latter should not be ruled out altogether. A simplified formulation more realistic than those identified in the investigation could be developed. One suggestion is to start with the Gabel et al. model [3] and extend it to the point where it can describe the passage of the blade through the ice using the scheme shown in Figure 1. This will require two minor modifications to the DND/MARTEC propeller analysis program: (i) provide a capability in the propeller geometry program for taking arbitrary slices through the blade, and (ii) extend the basic finite element solver to handle arbitrary pressure distributions. Actually, these modifications are necessary for any detailed finite element analysis of the ice block as well. Consequently, work on a more realistic simplified formulation of propeller/ice interaction forces would supplement the detailed finite element analysis. Furthermore, as work on the latter progresses, our application of the ice milling mechanisms will be enhanced, possibly allowing us to identify further improvements to the simplified model, which could, ultimately, provide an attractive alternative for preliminary designs.



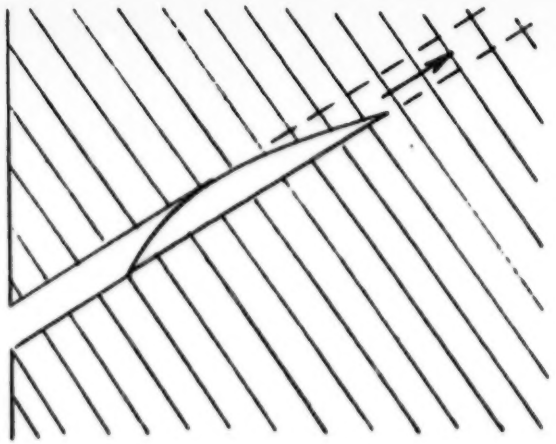
LOAD CASES:

#	θ
1	40°
2	50°
3	70°
4	90°
5	110°
6	130°
7	140°

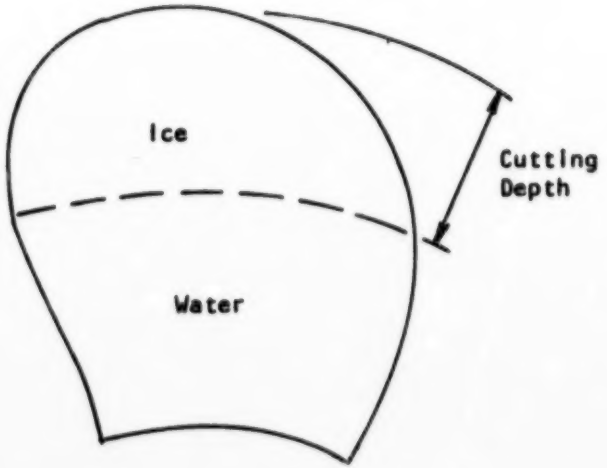
Figure 1: Ice Milling Model Used in Reference 2.

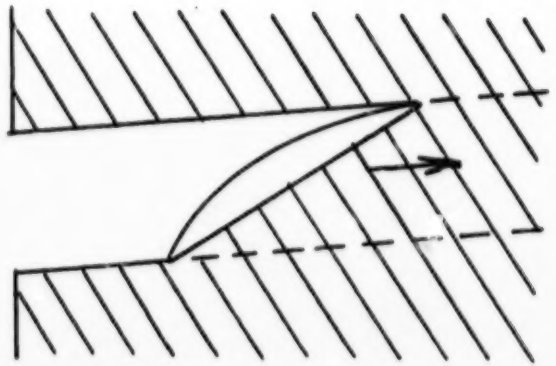


Shearing Mode



Slicing Mode





Crushing Mode

Figure 2: Ice Milling Model Used in Reference 3.

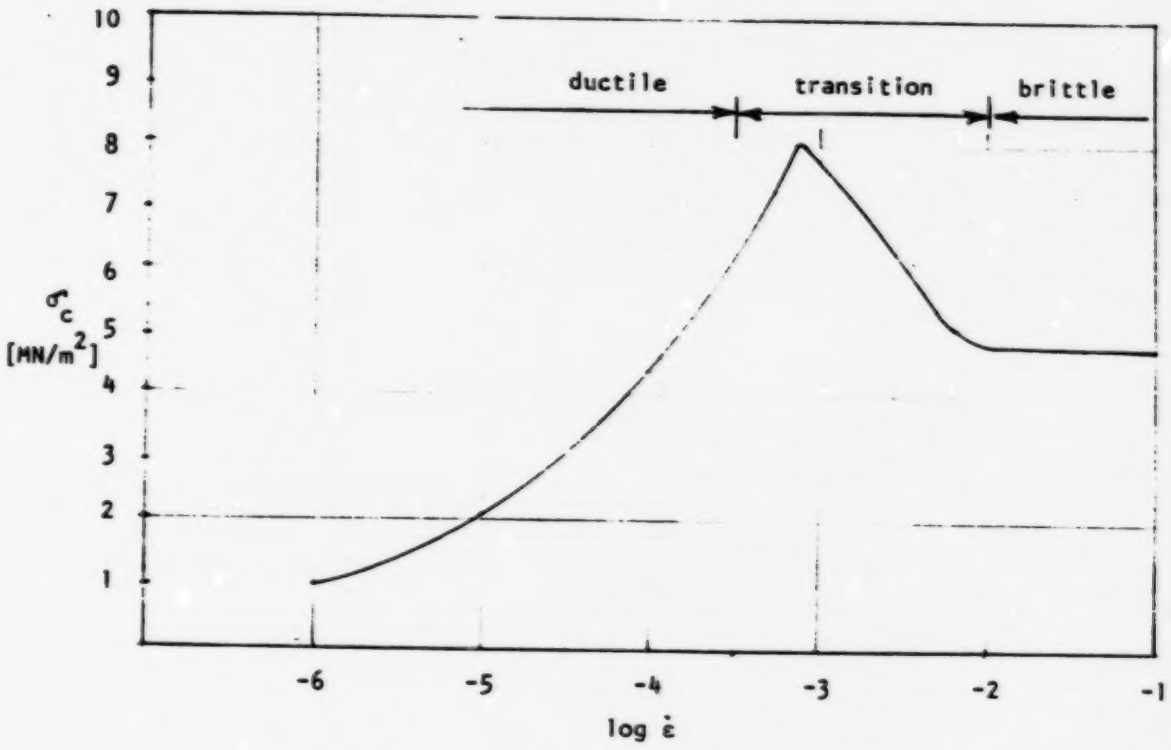


Figure 3: Crushing Strength of Ice Versus Strain-Rate [Ref. 92]

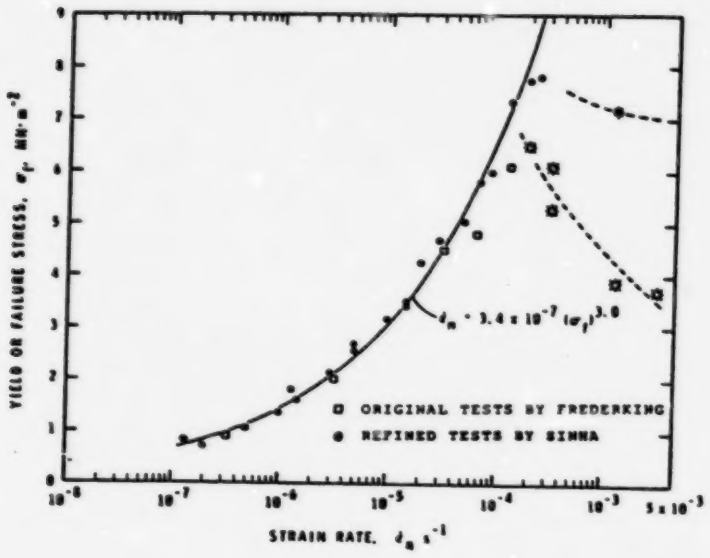


Figure 4: Dependence of Failure Stress on Nominal Strain Rate $\dot{\varepsilon}_n$ for Columnar-grained S_2-type Ice from the St. Lawrence and of Average Grain Size 4-5 mm at -10°C. Premature Failures are Marked with Asterisks. [Ref. 23].

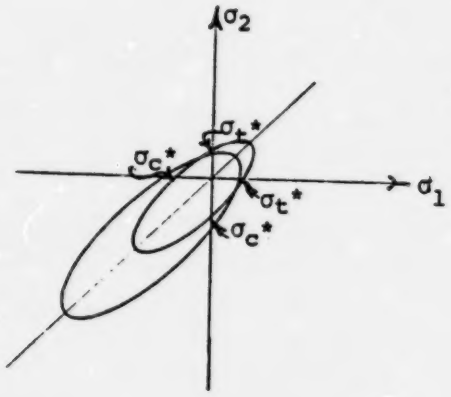


Figure 5: Sensitivity of Failure Envelope to Errors in Uniaxial Tensile and Compressive Strength Data.

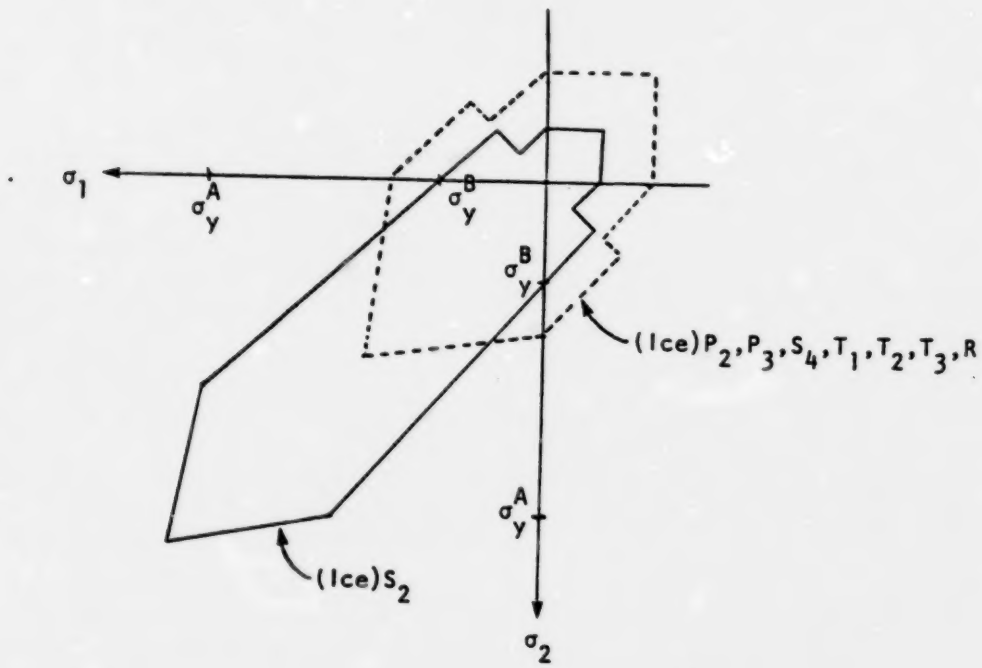


Figure 6: Proposed Failure Envelope for Both Columnar and Granular Ice Under Conditions of Plane Stresses [Ref. 30].

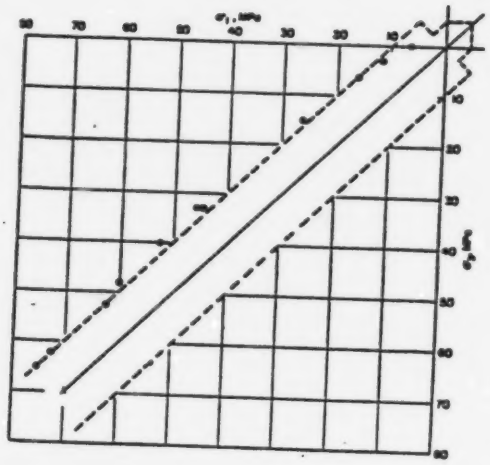


Figure 7: Proposed Failure Envelope for Fine Grained Ice of Glaciers Based on Triaxial Testing of Jones [Ref. 35].

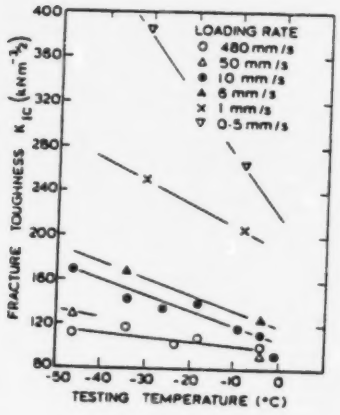


Figure 8: Fracture Toughness of Ice [Ref. 37].

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NOMENCLATURE

C	Cohesive strength of ice
d^*	Diameter of equivalent spherical crystal
E_{33}	Young's modulus along the c-axis of an ice crytal
G_{66}	Shear modulus in a plane perpendicular to the crystal c-axis
K	Stress intensity factor
K_c	Critical stress intensity factor associated with the onset of crack propagation
K_{Ia}	Stress intensity factor for arrest of plane strain tensile mode of crack extension
K_{Ic}	Stress intensity factor for commencement of plane strain tensile mode of crack extension
$\dot{N}$	Nucleation rate of material microfracture
R	Microfracture size
$\dot{R}$	Microfracture growth rate
α	Angle between normal to basal plane and loading direction
θ	Temperature
σ, σ_{ij}	Typical stress component
σ_c^*	Uniaxial compressive strength of ice
σ_{cv}	Surface energy of ice exposed to saturated vapor
σ_t^*	Uniaxial tensile strength of ice
σ_{no}, σ_{go}, σ_1, n	Material fracture parameters defining damage nucleation and growth laws
σ_1, σ_2, σ_3	Stress components along three coordinate directions

APPENDIX A

TRIP REPORTS

TRIP REPORT #1

Date: April 6, 1982

Location: Department of Civil Engineering
Ecole Polytechnique
P.O. Box 6079, Station A
Montreal, Quebec H3C 3A7

Present: Dr. René A. Tinawi, Ecole Polytechnique
Tel: (514) 344-4781, Ext. 4781

Dr. Michael Chernuka, Martec Limited

Dr. Tinawi conducts and supervises research in ice mechanics at Ecole Polytechnique. He also has extensive experience in the application of the finite element method to a wide range of engineering problems related to aeronautical structures, reinforced concrete as well as ice mechanics.

Dr. Tinawi advised strongly against attempting to develop a capability for full three dimensional analysis of the ice blocks under milling conditions at this point. The understanding of ice behaviour is too superficial and further fundamental research is still required. Furthermore, no previous application of analytical techniques such as the finite element method to ice milling type situations involving continuous material fracture has been reported in open literature and considerable development effort would be required to produce a reliable algorithm. It is very important that the feasibility of applying the developed analysis method to ice milling in two dimensions be demonstrated conclusively before attempting an analysis of three dimensional milling where the cost of both program development and execution will increase dramatically. Analytical results should probably be verified by comparision to two dimensional milling experiments which might not exist until appropriate tests are conducted.

Several possible sources of difficulty in algorithm development were identified:

i. Experience in ice experimentation at Ecole Polytechnique suggested that in dynamic problems, material strain rates decrease with distance from area of load application. Failure envelopes for several different strain rates are thus apparently required for the analysis of ice milling.

ii. A volume sufficiently large about the propeller-ice impact area must be analyzed so that applied boundary conditions have a negligible effect on stresses in the region of interest and thus on propeller pressure loads. No guidelines are available for establishing the extent of the required volume apriori.

iii. Proper representation of crushed ice requires clarification. Simply removing elements with crushed ice from analysis is unacceptable.

The possibility of conducting milling tests on laboratory facilities at Ecole Polytechnique in later phases of the propeller-ice interaction investigations was briefly discussed. Dr. Tinawi felt a study of fracture should be planned in connection with the milling tests. Fracture patterns generated during ice milling could be investigated in one of two ways. High speed films of milling tests could be examined frame by frame to assess fracture propagation. Alternately a step-by-step milling operation could be executed permitting tool removal after each increment of advance for detailed examination of fracture patterns within the block. While the second approach would obviously yield more information regarding ice fracture, reliability of the data collected would be questionable since generated fracture patterns are expected to depend on the rate of load application.

TRIP REPORT #2

Date: April 7, 1982

Location: Department of Civil Engineering
Facility of Sciences and Engineering
Pavilion Pouliot
Laval University
Quebec City G1K-7P4

Present: Professor Bernard Michel, Laval University
Tel: (418) 656-2205

Dr. Michael Chernuka, Martec Limited

For anyone involved in ice mechanics, Dr. Michel requires no introduction. His experimental research associated with the mechanical properties of ice and its interaction with engineering structures has lead to extensive publications including one of the few textbooks on the subject [1].

Dr. Michel was very interested in our problem of analytically modelling an ice block involved in propeller milling using the finite element method. There are resemblences between the ice indentation process studied by his research group and milling in its simplest forms. Also the finite element method [2] is being applied to some simple ice mechanics problems at Laval University by Gouri Dhatt.

An immediate appreciation of the complexity of the ice milling problem was demonstrated by Dr. Michel. He identified characteristics of the milling problem which may be difficult to model and offered some useful suggestions. One difficulty in particular, which he focused on and emphasized, was ice crushing by the blade and the subsequent extrusion of ice fragments from the blade path. Because of the high pressures associated with ice crushing, he felt the extrusion process represented a critical component of the milling process. Unless extrusion was appropriately accounted for in the numerical model, blade loads could not be accurately established.

Crack arrest and branching may provide additional difficulties if crack tracking is attempted. However Dr. Michel expects that the high strain rates associated with propeller ice interaction will result in ice fracture of the brittle type subject to a Tresca maximum shear type yield condition [3].

He strongly recommended performing controlled experiments involving a simplified milling situation preferably in two dimensions for the purposes of obtaining reliable data, comparisons with which would permit numerical methods developed for milling problems to be verified. A test configuration resembling that in Figure 1 might be considered since equipment used for indentation tests seem really adaptable for this purpose. Ideally the angle of attack should be variable. Pressure transducers may be

installed on the tool pressure surface in an attempt to establish the pressure distribution along the ice-tool interface. Shown at the bottom of Figure 1 are two different pressure distributions which might be obtained for =90 depending upon whether material extrusion occurs towards one or both ends of the ice-tool interface.

Although ice with granular microstructure can be reliably modelled using synthetic ice, polycrystalline ice, because of its anisotropic material properties, cannot. For this reason use of real ice would be preferred in milling experiments. Polycrystalline ice of type S_1 probably yields maximum milling loads and should be used in tests. S_1 ice is characterized by the c-axis of crystals being in parallel alignment. Since crystals form along the heat flow direction, the preferred orientation of the c-axis is vertical. Grains are usually large and of irregular shape. Crystal basal planes, however, are co-planar. As might be expected, special conditions are required to produce this type of ice and it is generally found in rivers, lakes and reservoirs where flow velocities are low. Apparently however, blocks of S_1 ice do occur within sheets of other types of ice and for this reason it is probably important to design icebreaker propellers to be capable of sustaining loads incurred in milling it. S_1 ice is very strong. Unlike isotropic ice where fracture occurs along the 45° shear plane for uniaxial tension or compression, fracture for S_1 ice is by shear along the basal plane irrespective of ice orientation with respect to the load direction except in the limiting situations of loading along or perpendicular to the basal plane in which other failure mechanisms are activated. The fracture process where maximum ice weakness is not aligned with direction of maximum shear stress is termed "kinkling". Shear strength of S_1 ice is typically 3922 KPa to 7846 KPa (40 kg/cm^2 - 80 kg/cm^2), tensile strength 2200 KPa to 4500 KPa and crushing strength 12000 KPa to 33000 KPa (see Figures 2 and 3).

I also briefly met with Jean-Paul Nadreau, formerly of Arctec Canada Ltd., who as a research student directed by Professor Michel, is investigating creep of Arctic ice and the definition of its triaxial failure envelope. After briefly describing his work, Mr. Nadreau showed me the excellent laboratory facilities operated by the ice mechanics group at Laval University. A total of three cold rooms are employed, one for ice generation (since temperature control is critical during ice generation, access to this cold room must be severely restricted for extended periods), another as a work room (for specimen preparation and ice testing) and the final one for indentation tests (containing a large rectangular tank, hydraulic loading ram and various measurement paraphernalia including displacement transdurers). In connection with their indentation investigations, the group pioneered a novel method of machining away the primary ice layer on the surface of the ice sheet.

A film of indentation tests performed at Laval University was viewed. Depending on the ratio of indenter width to ice thickness one of following four failure modes will occur: (i) shear, (ii) crushing and extrusion, (iii) plastification, or (iv) buckling.

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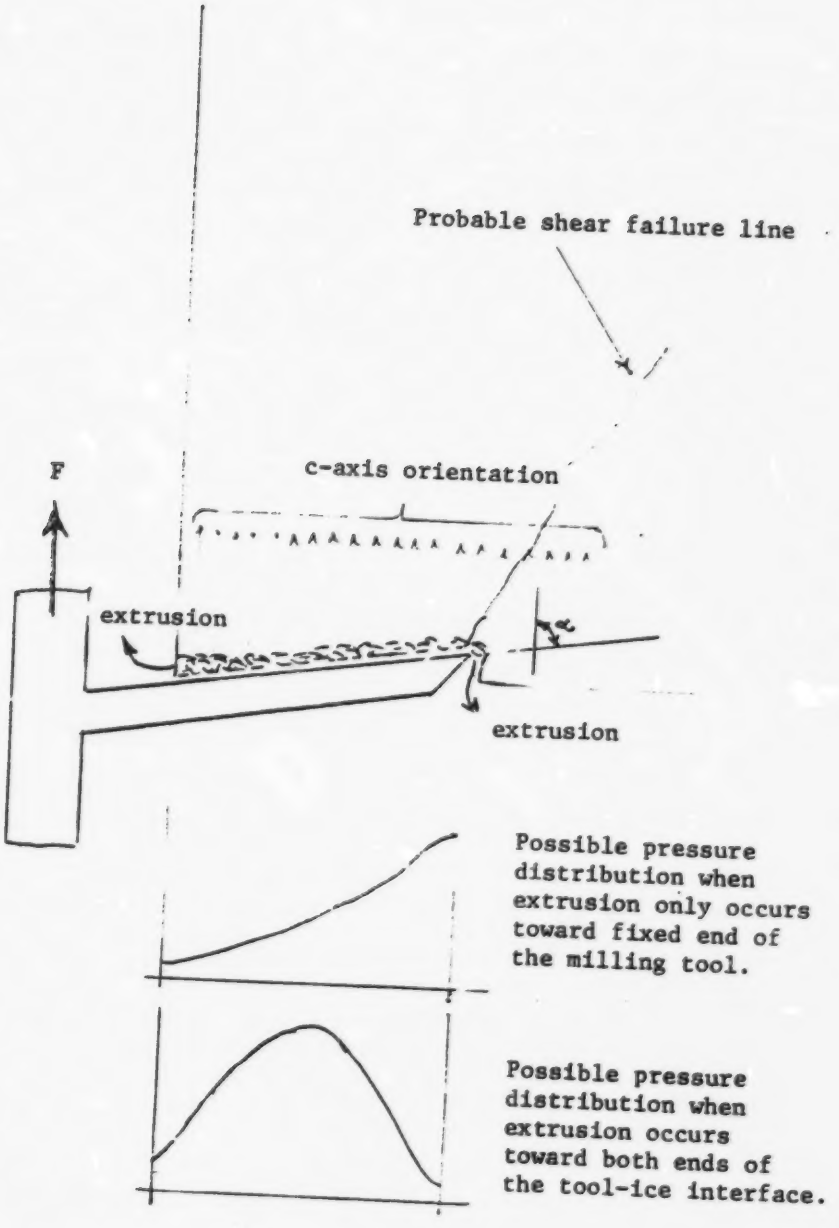
Probable shear failure line
F
c-axis orientation
extrusion
extrusion
Possible pressure distribution when extrusion only occurs toward fixed end of the milling tool.
Possible pressure distribution when extrusion occurs toward both ends of the tool-ice interface.

Fig. 1: Schematic diagram of an Idealized Milling Experiment with a Block of S_1 Ice.

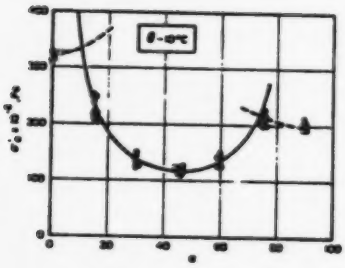

Fig. 2: Tensile Strength of S_1 Ice as a Function of Angle α Between the Normal to the Basal Plane and the Loading Direction [Ref. 3]

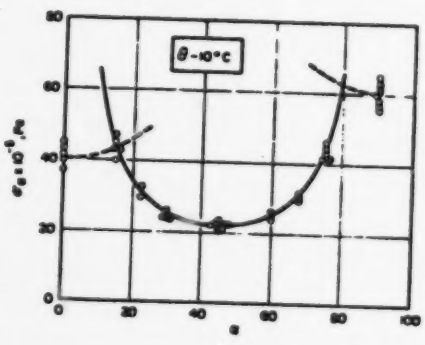

Fig. 3: Crushing Strength of S_1 Ice as a Function of Angle α Between the Normal to the Basal Plane and the Loading Direction [Ref. 3]

TRIP REPORT #3

Date: April 8, 1982

Location: Geotechnical Section
Division of Building Research
National Research Council
Ottawa, Ont. K1A 0R6

Present: Dr. Robert M. Frederking, NRC
Tel: (613) 993-1466

Dr. Michael Chernuka, Martec Limited

Considerable uncertainty in ice fracture predictions for multiaxial stress situations is due to the unreliability of the failure envelope definitions which arises from the scarcity of multiaxial ice strength measurements. Even in biaxial situation, testing has been limited to the plane strain tests of Frederking [1,2]. Various investigators have nonetheless generated two dimensional failure envelopes for ice by interpolating suitable curves to three or four uniaxial strength data points. However, two factors limit the accuracy of the envelopes thus generated. Typical uniaxial fracture strengths in compression, σ_c^* , are between two and three times the corresponding uniaxial fracture strength in tension, σ_t^*. Also a high degree of uncertainty generally exists in the σ_c^* and σ_t^* data points due to significant scatter in experimental data. The strong dependence of the ellipticity of the failure envelope on the data points used thus implies an even larger uncertainty in the shape of the interpolated failure envelope particularly along the major axis defined by $\sigma_1 = \sigma_2$ as demonstrated in Figure 1. Greater reliability should follow from the use of some biaxial strength data in envelope definition [2-5].

Triaxial yield envelopes pose even greater difficulties. Until very recently little triaxial strength testing was conducted and since those tests were performed on a standard triaxial cell, applied stress ratios were subject to the condition $\sigma_1 \neq \sigma_2 = \sigma_3$ [6,7]. Jones used this procedure to define the failure envelope of Figure 2 for fine grained ice of glaciers. Hydrostatic pressures of order 15 to 20 atmospheres encountered at glacier beds were considered.

A new multi-axial testing machine with a closed loop control system and brush type loading platens was recently used by F.U. Hausler [8] to perform more extensive triaxial ice strength tests. An increased number of stress ratios available through the above mentioned improvements in test equipment permitted a facetted failure surface to be defined. Although brush type platens were adopted with the aim of minimizing lateral strain constraints on the loaded ice faces, some concern has been expressed by Dr. N.K. Sinha that stress concentrations at the bristles induce premature cracking activity which reduces strength data obtained.

Arctec Canada Ltd. has performed a large amount of ice testing for a client. Although collected data is currently proprietary, under the

contractual arrangements public release is possible after December 1982. Apparently Transport Canada has obtained rights to the data but Dr. Frederking was unfamiliar with the conditions under which the rights were granted. Is the data to be used for one particular project? Can it be revealed to independent consultants employed under contract to Transport Canada? The Ship Safety Branch of Transport Canada should be approached for answers which would establish whether Martec is entitled to gain access.

Dr. Frederking expressed some reservations concerning certain proposals in literature regarding ice behaviour for high loading rates typical of those encountered in ice milling by propellers. While agreeing that ice is brittle for strain rates of order 10^{-2} sec^{-1} or 10^{-1} sec^{-1} in the sense that its response is elastic up to the point of failure, he is still not convinced that mechanical strengths and fracturing modes will remain unchanged as loading rates are further increased and inertial forces arising from material disposal assumed increased importance. Controlled experiments, although difficult to perform at high strain rates, are obviously necessary to ascertain the extend to which ice impact behaviour differs from brittle behaviour.

Potential sources of information relevant to ice milling were identified:

(i) Ray Ringer, Building M17. National Research Council Tel: 993-2439. Seven or eight years ago, the Low Temperature Laboratory performed some experiments on ice milling in which tool forces associated with ice cutting on lathes and milling machines were measured [9]. Of course, milling in water differs from milling in air and the results may be of limited value in predicting propeller forces developing from propeller-ice interaction.

(ii) APOA (Arctic Petroleum Operators Association) information exchange. This is a service for companies wishing to share with others information relating to Arctic operations. Available information is advertised in regularly updated bulletins. An interest in it can then be bought at a cost which is inversely proportional to the number of sharing companies. Project #80-2 may contain information relevant to ice milling by propellers. It refers to tests conducted by Sun Oil at Resolute Bay with a multitoothed rotary cutter in connection with the design of a self-propelled semi-submersible drilling rig for year-round use in the Canadian Arctic.

(iii) Malcolm Mellor of Cold Regions Research and Engineering Laboratory in Hanover, N.H. has conducted several investigations on machining of ice (for instance see CRREL Report 78-11). It was advised that we check whether BIO holds CRREL Reports or alternately contact Mr. Meller personally (603) 643-3200.

Both Dr. Frederking and Dr. Sinha of the Division of Building Research emphasized that whatever ice-propeller interaction model is adopted it should retain as much flexibility with respect to ice properties as possible. Even measured strength properties may change as testing procedures are refined.

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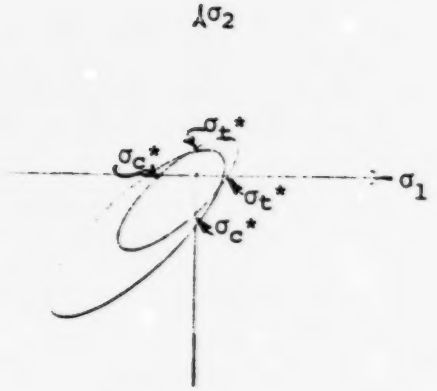

Figure 1: Sensitivity of Failure Envelope to Errors in Uniaxial Tensile and Compressive Strength Data.

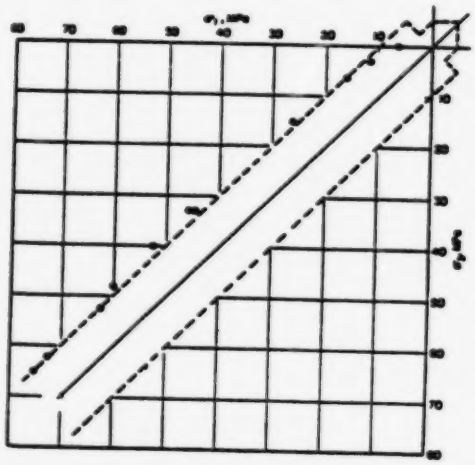

Figure 2: Proposed Failure Envelope for Fine Grained Ice of Glacier Based on Triaxial Testing of Jones [Ref. 7].

TRIP REPORT #4

Date: April 8, 1982

Location: Geotechnical Section
Division of Building Research
National Research Council of Canada
Ottawa, Ontario
K1A 0R6

Present: Dr. Nirmal K. Sinha, NRC
Tel: (613) 993-1466

Dr. Michael Chernuka, Martec Limited

Investigations undertaken by Dr. Sinha are aimed at achieving an improved understanding of ice behaviour and associated mechanisms as well as at developing systematic methods of testing and analyzing available ice strength data. Gradually as explanations are provided for all experimental observations, the basis for a predictive capability will be established in that behaviour yet unobserved could eventually be predicted with reasonable levels of confidence.

Important to both objectives is the notion of homologous temperatures equal to the non-dimensional ratio, T_w/T_m, where T_w is working temperature in ^{o}Kelvin and T_m is melting temperature in ^{o}Kelvin. Since ice temperatures are generally only slightly below freezing, homologous temperatures marginally less than 1.0 are common. Even at a working temperature of -10°C, T_w/T_m = .963. Ice behaviour for any given homologous temperature is similar to metals at the same homologous temperature. Recognizing this behaviour characteristic helps one to appreciate the wide range of mechanical properties normally displayed by ice including elasticity, viscoelasticity, creep rupture and brittle fracture and to formulate meaningful tests for the investigation of each.

Dr. Sinha questioned the decrease in uniaxial crushing strength for high loading rates frequently reported in literature. Observing that significant internal cracking activity and failure by brittle fracture accompanied high loading rates prompted him to investigate cracking activity occurring during testing. Failure strengths were found to be extremely susceptable to initial microcracking caused by specimen end roughness. Even small imperfections in the end finish produce stress concentrations which serve as crack initiation sites. Through refinements in techniques for specimen end finishing including smoothing with fine emery paper and polishing with alcohol-moistened tissue paper [1], significant increases in strength have been achieved so that only small strength reductions now accompany high loading rates as is evident from a comparison of the dashed lines on Figure 1.

Acoustic sensing devices employed during specimen testing provided further evidence to support Sinha's hypothesis regarding sensitivity of specimen strength to end imperfections. Two acoustical sensors approp-

riately attached to the specimen enabled the source of acoustic signals associated with cracking activity to be pinpointed accurately. As expected, signals were observed to originate predominately from platen ends during early stages of specimen loading. Dr. Sinha is currently investigating new specimen end configurations as a means of further reducing strength sensitivity to end imperfections when loading rates are high.

Dr. Sinha showed me the ice testing facilities at the Division of Building Research. Prompted by the hydrocarbon exploration in the Arctic, interest of the group has shifted from freshwater ice to sea ice and to meet changed testing requirements a new cold room was recently constructed. Extensive use has been made of stainless steel in construction to minimize corrosion. Situated in the new cold room is a new MTS testing machine characterized by a closed loop testing capability which eliminates testing machine compliance effects from strength measurements [2,3]. Collection of test data is computerized. A deep ice generation tank suitable for sea ice production is also located in the cold room. Thick insulation ensures thermal gradient is vertical and thus representative of the physical freezing in ice sheets. The reason for the increased tank depth is that as salt water freezes brine is expelled and the remaining water increases continuously in salinity. Typically sea water contains 35 parts per thousand of salt and ice just 4-7 parts per thousand. Obviously, therefore unless a significant depth of water remains at the bottom of the tank, salinity profile through the ice sheet will be significantly affected. At the time of the visit a supercooled tank of sea water was about to have ice formation precipitated by sprinkling some snow on its surface. Considerable testing is also done on naturally grown sea ice. While some field testing is performed as in the case of the Beaufort Sea [4], only limiting testing equipment is usually available. Consequently, numerous ice samples are collected on field trips to the Arctic and shipped to Ottawa for more extensive testing.

Dr. Sinha also outlined a replication process which he developed whereby thin sections of ice are etched and then coated in Formar (polyvinyl formal) to create a permanent record of surface microstructure for later detailed study (in the comfort of a heated room) [5].

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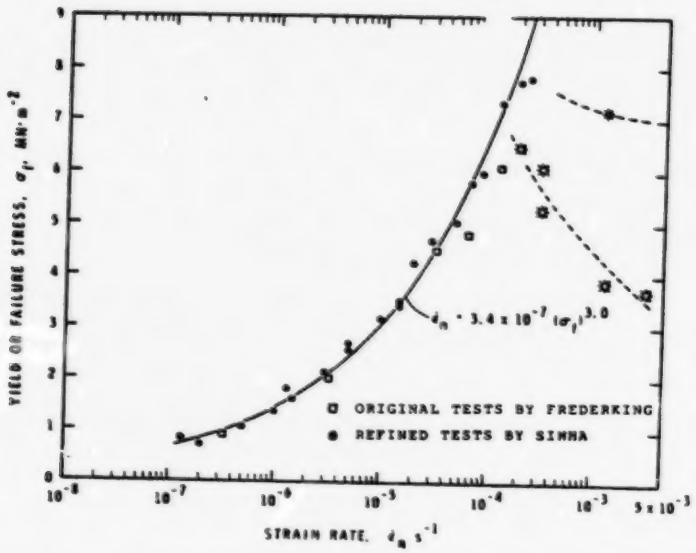
YIELD OR FAILURE STRESS, σ_f, MN·m^{-2}
STRAIN RATE, $\dot{\varepsilon}_n$ s^{-1}
$\dot{\varepsilon}_n = 3.4 \times 10^{-7} (\sigma_f)^{3.0}$
ORIGINAL TESTS BY FREDERKING
REFINED TESTS BY SINHA

Fig. 1: Dependence of Failure Stress on Nominal Strain Rate $\dot{\varepsilon}_n$ for Columnar-grained S_2-type Ice from the St. Lawrence and of Average Grain Size 4-5 mm at -10^0C. Premature Failures are Marked with Asterisks.[Ref. 1]

TRIP REPORT #5

Date: April 8, 1982

Location: Structures and Materials Laboratory - M13A
National Aeronautical Establishment
National Research Council
Ottawa, Ontario K1A 0R6

Present: Dr. Jerzy J. Kacprzynski, NRC
Tel: (613) 993-0289

Dr. Michael Chernuka, Martec Limited

Dr. Kacprzynski is an independent consultant to the National Research Establishment of the National Research Council of Canada whose duties include application of the finite element method to a large variety of problems originating primarily from research activities at the Structures and Materials Laboratory but also from technical matters referred to the laboratory by the Department of National Defence. For several years his research efforts concentrated on metal fracture and fatigue but recently emphasis has switched to the analysis of composite (layered) materials and structures.

Dr. Kacprzynski has used several commercial finite element programs either by rental from bureaus, such as the CDC Cybernet Service, or by outright purchase of source decks. This experience enabled him to outline the capabilities and limitations of a number of programs and to comment on their suitability for our ice milling application. Three programs in particular were recommended for further consideration.

(i) PISCES:- The program is based on the finite difference method for both space and time. Since required time steps are related to element size (i.e. time step must be small in comparison to time required for stress waves to propagate through an element), stability problems develop unless time steps are small. Consequently this program is recommended

(ii) MARC:- This program is based on the finite element method for space and the finite difference method for time. It is more suitable for longer time steps (and time intervals) since a different stability condition applies.

(iii) HEMP:- Developed by Lawrence Livermore Laboratory, it is also based on the finite element method for space and finite difference for time. The finite element formulation is of the Lagrangian type. Program HEMP is now superceded by DYNA2D and DYNA3D which incorporate an improved numerical integration scheme.

Dr. Kacprzynski believes PISCES combined with a damage model accounting for material cracking and fragmentation similar to that currently proposed for concrete by Bazant et al. [1] and for metals by

Rabotnov [2-4] represents the best approach. Stanford Research Institute has developed a powerful damage modelling computer program describing either ductile and brittle fracture associated with impact loadings [5-7] which can be readily inserted in wave propagation programs such as HEMP and PISCES. Their model accounts for nucleation and subsequent growth of cracks or voids as a function of stress level and stress duration, coalescence of cracks to form fragments, and the possible eventual separation of fragments. The effects of inherent material flaws and faults can also be accounted for. The program has been refined to the point that size of fragments can be reliably predicted. The cost of purchasing and implementing this material damage model into PISCES was quoted two years ago at $10,000 U.S. Expressing a concern about the magnitude of this start up cost to our local CDC Cybernet representative recently, I was informed that CDC may be willing to assume it with intentions presumably of recovering it through royalty levies on all users. (Dr. Kacprzynski for instance would be interested in this enhanced PISCES program for analysis of damage in regions of plastic yielding at crack-tips under fatique conditions).

Unfortunately no documentation on the PISCES program was available for examination at the time of my visit. I had hoped to review program input requirements and material properties in particular. In the course of my discussion with Dr. Kacprzynski for instance I learned that PISCES requires spalling pressure as input; this is probably unknown paramater for ice.

Dr. Kacprzynski identified several references on fracture and crack propagation which might be of interest [8-12].

An ice research group at the University of Manitoba directed by Professor H. Cohen is developing analytical methods of estimating bearing capacity of floating ice sheets under contract to the Canadian Transport Department [13]. Contact with them should probably be established.

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